Aufgab 3 [7 Pumkte]

Lösen Sie folgende inhomogene DGL

$$y' = \frac{y}{1+x^2} + 2x - 1 \qquad y(0)=0$$

Hinweis: Lösen Sie zuerst homogene DGL: $y' = \frac{y}{1+x^2}$

$$(f\cdot g)' = f'\cdot g + f\cdot g'$$

$$\left(e^{C(x)}\cdot y(x)\right)' =$$

$$= e^{C(x)}\cdot C'(x)\cdot y(x) + e^{C(x)}\cdot y'(x)$$

$$= \left(y'(x) + C'(x)\cdot y(x)\right)\cdot e^{C(x)}$$

$$\Rightarrow C'(x) = -\frac{1}{1+x^2}$$

$$C(x) = -\int \frac{1}{1+x^2}\,dx = -\arctan(x)$$

$$y' = \frac{y}{1+x^2} + 2x - 1$$

$$y' - \frac{y}{1+x^2} = 2x-1 \qquad \Big|\cdot e^{C(x)}$$

$$\left(y' - \frac{y}{1+x^2}\right)\cdot e^{C(x)} = (2x-1)\cdot e^{C(x)}$$

$$\left(e^{C(x)}\cdot y(x)\right)' = (2x-1)\cdot e^{C(x)}$$

$$\Big(\underbrace{e^{-\arctan(x)}\cdot y(x)}_{=:z(x)}\Big)' = (2x-1)\cdot e^{-\arctan(x)}$$

$$z'(x) = (2x-1)\,e^{-\arctan(x)}$$

$$z(0) = e^{-\arctan(0)}\cdot \underbrace{y(0)}_{=0} = 0$$

$$\boxed{\begin{array}{l} z'(x) = f(x),\quad z(x_0) = z_0 \\ \text{Lösung:} \\ z(x) = z_0 + \int_{x_0}^{x} f(t)\,dt \end{array}}$$

$$\Rightarrow z(x) = \underbrace{z(0)}_{=0} + \int_0^x (2t-1)\cdot e^{-\arctan(t)}\,dt$$

$$= \int_0^x (2t-1)\,e^{-\arctan(t)}\,dt$$

$$z(x) = e^{-\arctan(x)}\cdot y(x)$$

$$\Rightarrow y(x) = z(x)\cdot e^{\arctan(x)} = \int_0^x (2t-1)\,e^{-\arctan(t)}\,dt \cdot e^{\arctan(x)}$$