$(f\cdot g)' = f'\cdot g + f\cdot g'$

$\left(e^{-2x}\cdot y(x)\right)' =$

$= e^{-2x}\cdot(-2)\cdot y(x) + e^{-2x}\cdot y'(x)$

$= \left(y'(x) - 2y(x)\right)\cdot e^{-2x}$

Hauptsatz

$f:\mathbb{R}\to\mathbb{R}$ stetig

$x_0\in\mathbb{R}$

$F:\mathbb{R}\to\mathbb{R}_x$

$x\mapsto\int_{x_0}^{x} f(t)\,dt$

Dann gilt $F'(x) = f(x)$

b) $y' - 2y = (x+1)^2$, $y(0) = 1$

Variation der Konstanten:

$y'(x) - 2y(x) = (x+1)^2 \quad | \cdot e^{-2x}$

$\underbrace{(y'(x) - 2y(x))\cdot e^{-2x}} = (x+1)^2\cdot e^{-2x}$

$(\underbrace{e^{-2x}\cdot y(x)}_{=:z(x)})' = (x+1)^2\cdot e^{-2x}$

$z'(x) = (x+1)^2\cdot e^{-2x}$, $z(0) = e^{-2\cdot 0}\cdot y(0) = \underbrace{e^0}_{1}\cdot 1 = 1$

$\Rightarrow$

$z(x) = \underbrace{z(0)}_{1} + \int_0^x (t+1)^2\cdot e^{-2t}\,dt$

$z(0) = 1 + \underbrace{\int_0^0 \dots\,dt}_{=0} = 1$ ✓

$z'(x) = \left(\int_0^x (t+1)^2\cdot e^{-2t}\right)' = (x+1)^2\cdot e^{-2x}$

$\int_a^b f(t)g'(t)\,dt = \left[f(t)g(t)\right]_a^b - \int_a^b f'(t)g(t)\,dt$

$\int_0^x \underbrace{(t+1)^2}_{f}\cdot\underbrace{e^{-2t}}_{g'}\,dt = \left[(t+1)^2\cdot\left(-\tfrac{1}{2}\right)e^{-2t}\right]_0^x - \int_0^x 2(t+1)\cdot\left(-\tfrac{1}{2}\right)\cdot e^{-2t}\,dt =$

$= \left(-\tfrac{1}{2}\right)(x+1)^2\cdot e^{-2x} + \tfrac{1}{2} + \int_0^x \underbrace{(t+1)}_{f}\,\underbrace{e^{-2t}}_{g'}\,dt =$

$(-\tfrac{1}{2}e^{-2t})' = \cancel{(-\tfrac{1}{2})}e^{-2t}\cdot\cancel{(-2)} = e^{-2t}$

$= \quad '' \quad + \quad '' \quad + \left[(t+1)\left(-\tfrac{1}{2}\right)e^{-2t}\right]_0^x - \underbrace{\int_0^x 1\cdot\left(-\tfrac{1}{2}\right)e^{-2t}\,dt}_{=\left[\frac{1}{4}e^{-2t}\right]_0^x = \frac{1}{4}(e^{-2x}-1)}$

$\boxed{\left[F(x)\right]_a^b = F(b) - F(a)}$

$= \left(-\tfrac{1}{2}\right)(x+1)^2 e^{-2x} + \tfrac{1}{2} - \tfrac{1}{2}(x+1)e^{-2x} + \tfrac{1}{2} + \tfrac{1}{4}\left(1 - e^{-2x}\right)$

Aufgabe 5 [7+6 *Punkte*]

Lösen Sie folgende Differentialgleichungen:

a) $y'' - 6y' + 9y = e^{3t}$ $\quad y(0)=1,\ y'(0)=0$

b) $y' - 2y = (x+1)^2$ $\quad y(0)=1$

a) Ansatz: $y(t) = A\cdot e^{3t}$

$y'(t) = A\cdot e^{3t}\cdot 3 = 3Ae^{3t}$

$y''(t) = (y'(t))' = 3A\cdot e^{3t}\cdot 3$

$= 9Ae^{3t}$

$y'' - 6y' + 9y = 0$

$\underbrace{9Ae^{3t} - 18Ae^{3t} + 9Ae^{3t}}_{=0} = 0$

$0 = 0$

$y(t) = A\cdot e^{3t}$ löst $y'' - 6y' + 9y = 0$

$(a\cdot f(x))' =$

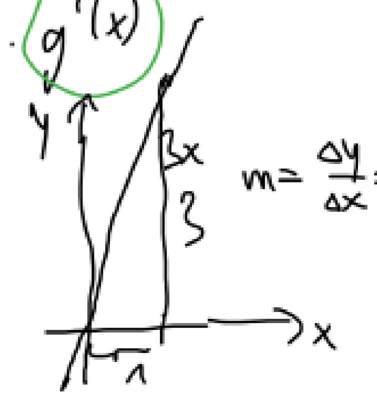
$= a\cdot f'(x)$

$(e^{3x})' = e^{3x}\cdot(3x)'$

$= e^{3x}\cdot 3$

$(f(g(x)))' = f'(g(x))\cdot g'(x)$

$(x^3)' = 3x^2$



$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$

$(\underbrace{e^{-2x}\cdot y(x)}_{z(x)})' = (x+1)^2\cdot e^{-2x}$

$5^{-3} = \frac{1}{5^3}$

$z(x) = e^{-2x}\cdot y(x)$

$\Rightarrow y(x) = \frac{z(x)}{e^{-2x}} = z(x)\cdot e^{2x} =$

$= \left(-\tfrac{1}{2}\right)(x+1)^2 + e^{2x} - \tfrac{1}{2}(x+1) + \tfrac{1}{4}e^{2x} - \tfrac{1}{4}$

$e^{-2x}\cdot e^{2x} = 1$