b) $u'(t) = 2(1+t)\cdot e^{-u(t)}$ mit $\overset{t_0}{u(0)} = \overset{u_0}{0}$ $\quad | \cdot e^{u(t)}$

Hauptsatz

$f: \mathbb{R} \to \mathbb{R}$ stetig und $x_0 \in \mathbb{R}$, $y_0 \in \mathbb{R}$

$F: \mathbb{R} \to \mathbb{R}$

$x \mapsto \int_{x_0}^{x} f(t)\,dt + y_0$

Dann gilt:

$F'(x) = f(x)$ und $F(x_0) = \underbrace{\int_{x_0}^{x_0} f(t)\,dt}_{=0} + y_0 = y_0$

Beispiel:

$$\left(\int_5^x \sin(t^3)\,dt\right)' = \sin(x^3)$$

$$\left(\int_{-7}^x t^5 + e^{t^2}\,dt\right)' = x^5 + e^{x^2}$$

$$\left(\int_{-100}^x \tan(\sin(t^3))\,dt\right)' = \tan(\sin(x^3))$$

$$\boxed{(f(g(x)))' = f'(g(x)) \cdot g'(x) \quad \text{Kettenregel}}$$

$$\left(\int_{-5}^{x^3} \sin(e^t)\,dt\right)' = \sin(e^{x^3}) \cdot 3\cdot x^2$$

$$\left(\int_{-7}^{x^5} e^{\sin(t^2)}\,dt\right)' = e^{\sin(x^{10})} \cdot 5\cdot x^4$$

$$\left(\int_{3}^{x^7} \cos(t^2+t)\,dt\right)' = \cos(x^{14}+x^7)\cdot 7\cdot x^6$$

$$\left(\int_{3}^{u(x)} s^2\,ds\right)' = (u(x))^2 \cdot u'(x)$$

$$u'(t)\cdot e^{u(t)} = 2(1+t)$$

$$\left(\int_{u_0}^{u(t)} e^s\,ds\right)' = e^{u(t)}\cdot u'(t)$$

$$\int_{u_0}^{u(t)} e^s\,ds = \int_{t_0}^{t} 2\cdot(1+s)\,ds$$

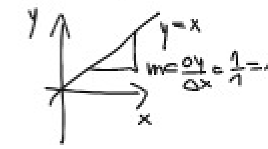
$u_0 = 0$, $t_0 = 0$

$$[e^s]_0^{u(t)} = e^{u(t)} - 1$$

$$2\cdot\left[s + \tfrac{1}{2}s^2\right]_0^t = 2\cdot\left(t + \tfrac{1}{2}t^2 - 0\right)$$

$$[F(x)]_a^b = F(b) - F(a)$$

$(x)' = 1$



$5 = e^x \quad | \ln(\,)$

$\ln(5) = x$

$5 = e^x \quad | \cdot e^{-x}$

$5\cdot e^{-x} = 1$

$$\Rightarrow e^{u(t)} - 1 = 2\cdot\left(t + \tfrac{1}{2}t^2\right) \quad | +1$$

$$e^{u(t)} = 2t + t^2 + 1 \quad | \ln(\,)$$

$$u(t) = \ln(t^2 + 2t + 1)$$

$$= \ln((t+1)^2) = 2\cdot\ln(t+1)$$

$$\boxed{\ln(a^b) = b\cdot\ln(a)}$$

b) $\underbrace{u'(t)}_{LS} = \underbrace{2(1+t)\cdot e^{-u(t)}}_{RS}$ mit $u(0) = 0$

$\ln(e^x) = x$

Probe: $u(0) = 2\cdot\ln(0+1) = 2\cdot\ln(1) = 2\cdot\ln(e^0) = 2\cdot 0 = 0$ ✓

$$LS = u'(t) = (2\cdot\ln(t+1))' = 2\cdot(\ln(t+1))' = 2\cdot\frac{1}{t+1}$$

$$\boxed{(a\cdot f(x))' = a\cdot f'(x)}$$

$$RS = 2\cdot(1+t)\cdot e^{-u(t)} =$$

$$= 2\cdot(1+t)\cdot e^{-2\cdot\ln(t+1)}$$

$$= 2\cdot(1+t)\cdot e^{\ln((t+1)^{-2})} =$$

$$\boxed{\ln(a^b) = b\cdot\ln(a)}$$

$$= 2\cdot(1+t)\cdot e^{\ln\left(\frac{1}{(t+1)^2}\right)} =$$

$$\boxed{e^{\ln(x)} = x \quad \forall x \in \mathbb{R}}$$

$$= 2\cdot(1+t)\cdot\frac{1}{(t+1)^2} = 2\cdot\frac{1}{t+1} = LS \ ✓$$

miro

Lösen Sie die folgenden Differentialgleichungen 1. Ordnung:

Aufgabe 1: $y' = x + 1, \quad y(-2) = -1$

$\boxed{\text{Aufgabe 2: } y' = 0.5\,(3 - y), \quad y(\underset{=x_0}{0}) = 2 = y_0}$

Aufgabe 3: $y' = y - 5, \quad 1)\ y(0) = 2, \quad 2)\ y(1) = -2$

Aufgabe 4: $y' = y^2 \sin x$

$y_1(0) = 1, \quad y_2(0) = \frac{1}{2}, \quad y_3(0) = -2$

$$y'(x) = \frac{1}{2} \cdot (3 - y(x)) \quad | : (3 - y(x))$$

$$y'(x) \cdot \frac{1}{3 - y(x)} = \frac{1}{2}$$

$$\int_{2=y_0}^{y(x)} \frac{1}{3-s}\,ds = \int_{0=x_0}^{x} \frac{1}{2}\,ds$$

$$(\ln(3-s))' = \frac{1}{3-s} \cdot (-1)$$

$$\Big[(-1) \cdot \ln(3-s)\Big]_2^{y(x)} \qquad \Big[\frac{1}{2}s\Big]_0^x$$

$$-\ln(3 - y(x)) + \underbrace{\ln(1)}_{=0} \qquad \frac{1}{2}x$$

$$\boxed{e^{\ln(x)} = x \quad \forall x > 0}$$

$$\Rightarrow \quad -\ln(3 - y(x)) = \frac{1}{2}x \quad | \cdot (-1)$$

$$\ln(3 - y(x)) = -\frac{1}{2}x \quad | e^{(\,)}$$

$$\ln(x) = 5 \quad | e^{()}$$
$$x = e^5$$

$$3 - y(x) = e^{-\frac{1}{2}x} \quad | -3$$

$$-y(x) = e^{-\frac{1}{2}x} - 3 \quad | \cdot (-1)$$

$$y(x) = 3 - e^{-\frac{1}{2}x}$$

Aufgabe 2: $\underbrace{y'}_{LS} = \underbrace{0.5\,(3 - y)}_{RS}, \quad y(0) = 2$

Probe: $y(0) = 3 - e^{-\frac{1}{2}\cdot 0} = 3 - 1 = 2$ ✓

$$LS = y'(x) = \left(3 - e^{-\frac{1}{2}x}\right)' = (-1) \cdot e^{-\frac{1}{2}x} \cdot \left(-\frac{1}{2}\right) = \frac{1}{2} e^{-\frac{1}{2}x}$$

$$RS = \frac{1}{2} \cdot \left(3 - \left(3 - e^{-\frac{1}{2}x}\right)\right) =$$

$$(e^{5x})' = e^{5x} \cdot 5$$

$$= \frac{1}{2} \cdot \left(\underbrace{3 - 3}_{=0} + e^{-\frac{1}{2}x}\right) = \frac{1}{2} e^{-\frac{1}{2}x} = LS$$ ✓

miro

Aufgabe 4: $y' = y^2 \sin x$

$y_1(0) = 1, \quad \cancel{y_2(0) = \frac{1}{2}, \quad y_3(0) = -2}$

$$y'(x) = y^2(x) \cdot \sin x \qquad \Big| : y^2(x)$$

Trennung der Variablen

$$y'(x) \cdot \frac{1}{y^2(x)} = \sin x$$

$$\int_{1 = y_0}^{y(x)} \frac{1}{s^2}\, ds = \int_{0 = x_0}^{x} \sin(s)\, ds$$

$$\left[-\frac{1}{s} \right]_1^{y(x)} \qquad\qquad \left[-\cos(s) \right]_0^{x}$$

$$-\frac{1}{y(x)} + 1 \qquad\qquad -\cos(x) + \underbrace{\cos(0)}_{=1}$$

$$\Rightarrow \; -\frac{1}{y(x)} + 1 = -\cos(x) + 1$$

$$(\ln(s^2))' = \frac{1}{s^2} \cdot 2 \cdot s$$

$$\int x^n\, dx = \frac{1}{n+1} x^{n+1}$$

$$\int \frac{1}{s^2}\, ds = \int s^{-2}\, ds = (-1)\, s^{-1} = -\frac{1}{s}$$