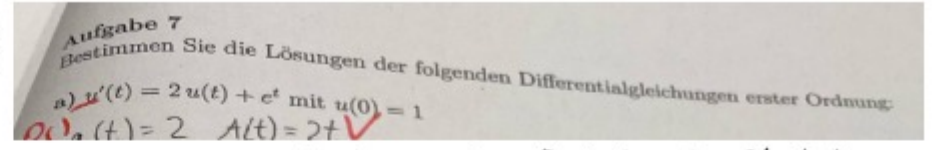


Separation der Variablen
 $y'(x) = f(x) \cdot g(y(x)) \quad | :g(y(x))$
 $\frac{y'(x)}{g(y(x))} = f(x)$



Variation der Konstanten
 $u'(t) = 2u(t) + e^t \quad | -2 \cdot u(t)$
 $u'(t) - 2 \cdot u(t) = e^t \quad | \cdot e^{-2t}$
 $(u'(t) - 2 \cdot u(t)) \cdot e^{-2t} = e^t \cdot e^{-2t}$
 $(u(t) \cdot e^{-2t})' = e^t \cdot e^{-2t}$
 $=: z(t)$

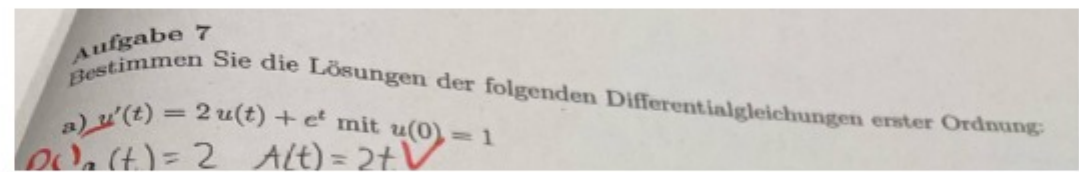
Sei A Stammfkt von a , d.h. $A' = a$
 $(f \cdot g)' = f' \cdot g + f \cdot g'$
 $(y(t) \cdot e^{A(t)})' = y'(t) \cdot e^{A(t)} + y(t) \cdot \underbrace{A'(t)}_{=a(t)} \cdot e^{A(t)}$
 $= (y'(t) + a(t) \cdot y(t)) \cdot e^{A(t)}$

Produktregel
 $(f \cdot g)' = f' \cdot g + f \cdot g'$
 $(u(t) \cdot e^{-2t})' = u'(t) \cdot e^{-2t} + u(t) \cdot (-2) \cdot e^{-2t}$
 $= (u'(t) - 2u(t)) \cdot e^{-2t}$

$z'(t) = e^{-t}$
 $z(0) = u(0) \cdot e^{-2 \cdot 0} = 1 \cdot 1 = 1 =: z_0$
 $\Rightarrow z(t) = z_0 + \int_{t_0=0}^t e^{-s} ds = 1 + [-e^{-s}]_0^t = 1 - e^{-t} + 1 = 2 - e^{-t}$

Hauptsatz 2
 $f: \mathbb{R} \rightarrow \mathbb{R}, x_0, y_0 \in \mathbb{R}$
 $F: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto y_0 + \int_{x_0}^x f(s) ds$
 Dann gilt $F' = f$
 und $F(x_0) = y_0 + \int_{x_0}^{x_0} f(s) ds = y_0$

$z(t) = u(t) \cdot e^{-2t} \quad | \cdot e^{2t}$
 $\Rightarrow u(t) = z(t) \cdot e^{2t} = (2 - e^{-t}) \cdot e^{2t} = 2e^{2t} - e^t$



Probe:
 $u(0) = 2 \cdot e^{2 \cdot 0} - e^0 = 2 - 1 = 1 \checkmark$
 $u'(t) = (2e^{2t} - e^t)' = 4e^{2t} - e^t$
 $2u(t) + e^t = 2 \cdot (2e^{2t} - e^t) + e^t = 4e^{2t} - 2e^t + e^t = 4e^{2t} - e^t$
 $\Rightarrow u'(t) = 2u(t) + e^t$

$$\int \frac{1}{x} dx = \ln(x)$$

$$a(x) = \frac{1}{1+x}$$

$$A(x) = \int \frac{1}{1+x} dx = \ln(1+x)$$

$$(\ln(1+x))' = \frac{1}{1+x} \cdot \underbrace{(1+x)'}_{=1}$$

5. Bestimmen Sie die allgemeine Lösung durch Variation der Konstanten
 (a) $y' + \frac{y}{1+x} = e^{2x}$ $y(0) = 1$
 (b) $y' \cos(x) - y \sin(x) = 1$ $y(0) = 1$

$$y'(x) + \frac{y}{1+x} = e^{2x}$$

$$y'(x) + \frac{1}{1+x} \cdot y(x) = e^{2x} \quad \left| \cdot e^{\int \frac{1}{1+x} dx} = e^{\ln(1+x)} = 1+x \right.$$

$$(y'(x) + \frac{1}{1+x} \cdot y(x)) \cdot (x+1) = e^{2x} \cdot (x+1)$$

$$\left(\underbrace{y(x) \cdot (x+1)}_{z(x)} \right)' = e^{2x} \cdot (x+1)$$

$$z(0) = y(0) \cdot (0+1) = 1 \cdot 1 = 1$$

$$z'(x) = e^{2x} \cdot (x+1)$$

$$\Rightarrow z(x) = 1 + \int_0^x e^{2s} \cdot (s+1) ds$$

$$= 1 + \left[\frac{1}{2} e^{2s} \cdot (s+1) \right]_0^x - \int_0^x \frac{1}{2} e^{2s} \cdot 1 ds = 1 + \frac{1}{2} e^{2x} \cdot (x+1) - \frac{1}{2} - \frac{1}{2} \left[\frac{1}{2} e^{2s} \right]_0^x$$

$$= \frac{1}{2} + \frac{1}{2} e^{2x} (x+1) - \frac{1}{4} (e^{2x} - 1)$$

Partielle Integration

$$\int_a^b g'(x) \cdot f(x) dx = [g(x) \cdot f(x)]_a^b - \int_a^b g(x) \cdot f'(x) dx$$

$$= \frac{1}{2} + \frac{1}{2} e^{2x} \cdot (x+1) - \frac{1}{2} \cdot \frac{1}{2} e^{2x} + \frac{1}{4}$$

$$= \frac{3}{4} + \frac{1}{2} e^{2x} \left(x + \frac{1}{2} \right)$$

$$z(x) = y(x) \cdot (x+1)$$

$$\Rightarrow y(x) = \frac{z(x)}{x+1} = \frac{\frac{3}{4} + \frac{1}{2} e^{2x} \left(x + \frac{1}{2} \right)}{x+1}$$

$$y(0) = \frac{\frac{3}{4} + \frac{1}{2} e^{2 \cdot 0} \cdot \left(0 + \frac{1}{2} \right)}{0+1} = \frac{\frac{3}{4} + \frac{1}{2} \cdot 1 \cdot \frac{1}{2}}{1} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \quad \checkmark$$

5. Bestimmen Sie die allgemeine Lösung durch Variation der Konstanten

(a) $y' + \frac{y}{1+x} = e^{2x}$ $y(0) = 1$

(b) $y' \cos(x) - y \sin(x) = 1$ $y(0) = 1$

$$y' + \frac{y}{1+x} =$$

$$y'(x) = \left(\frac{\frac{3}{4} + \frac{1}{2} e^{2x} \left(x + \frac{1}{2} \right)}{x+1} \right)'$$

$$\left(\frac{f}{g} \right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$