$\left(\int_5^x \sin(t^2)\,dt\right)' = \sin(x^2)$

$\left(\int_{100}^x t^3 \cdot \cos(t)\,dt\right)' = x^3 \cdot \cos(x)$

Hauptsatz der Differential- und Integralrechnung

$f: \mathbb{R} \to \mathbb{R}$ stetig, $x_0 \in \mathbb{R}$

$F: \mathbb{R} \to \mathbb{R}$, $x \mapsto \int_{x_0}^x f(s)\,ds \Rightarrow F'(x) = f(x)$

$g'(x) = \frac{1}{x}$ $\quad g(5) = 4$

$\Rightarrow g(x) = 4 + \int_5^x g'(s)\,ds$

$f'(z) = \frac{1}{z}$ $\quad f(1) = 2\pi i$

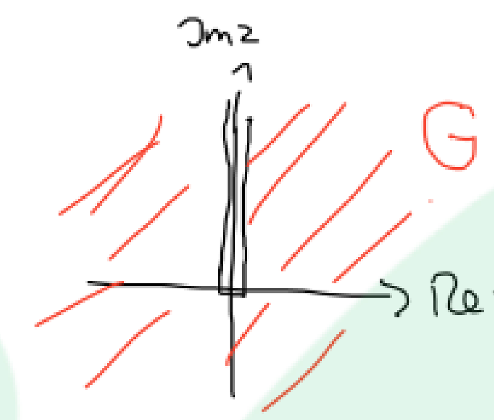
$f(z) = 2\pi i + \int_\gamma \underbrace{f'(z)}_{=\frac{1}{z}}\,dz$

wobei γ Weg mit Start 1 und Ende z mit Spur $\gamma \subseteq G$

$\gamma: [0,1] \to \mathbb{C}$

$\int_\gamma g(z)\,dz = \int_0^1 g(\gamma(t)) \cdot \gamma'(t)\,dt$

b)



$f\left(\frac{1}{e^2}\right) = 2\pi i + \int_\gamma \frac{1}{z}\,dz = 2\pi i + \int_0^1 \frac{1}{\gamma(t)} \gamma'(t)\,dt =$

$= 2\pi i + \int_0^1 \cancel{e^{2t}} (-2) \cdot \cancel{e^{-2t}}\,dt =$

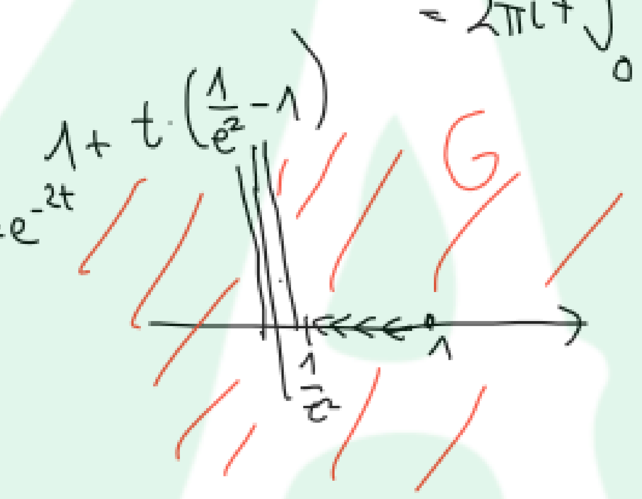
$= 2\pi i + (-2) \underbrace{\int_0^1 1\,dt}_{=1}$

$= 2\pi i - 2 = \underline{\underline{2(\pi i - 1)}}$

$\gamma: [0,1] \to G$

$t \mapsto \frac{1}{e^{2t}} = e^{-2t}$

$1 + t\left(\frac{1}{e^2} - 1\right)$



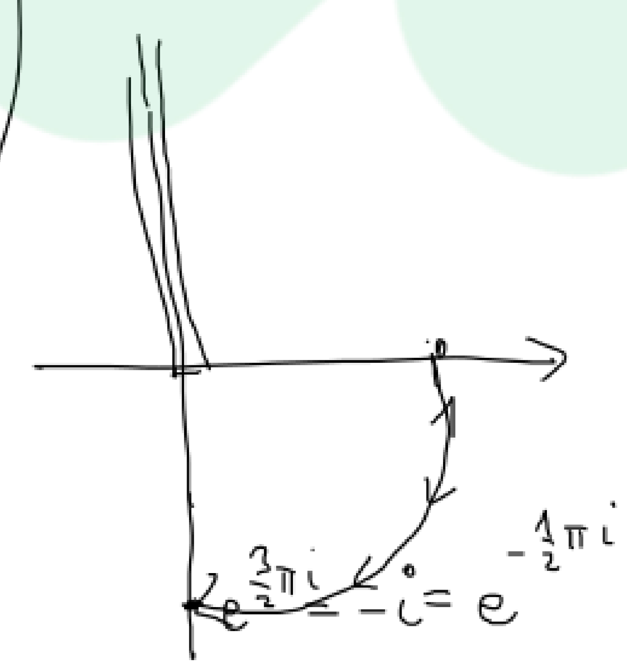
$f\left(\frac{1}{e^2}\right) = 2(\pi i - 1)$

$f\left(\underbrace{e^{\frac{3}{2}\pi i}}_{=-i}\right) = 2\pi i + \int_\gamma \frac{1}{z}\,dz =$ wobei

γ Weg mit Spur $\gamma \subseteq G$ und Start 1 und Ende $e^{\frac{3}{2}\pi i}$

$\gamma: [0,1] \to G$

$t \mapsto e^{-\frac{1}{2}\pi i t}$



$= 2\pi i + \int_0^1 \frac{1}{\gamma(t)} \cdot \gamma'(t)\,dt =$

$= 2\pi i + \int_0^1 \cancel{e^{\frac{1}{2}\pi i t}} \cdot \left(-\frac{1}{2}\right)\pi i \cdot \cancel{e^{-\frac{1}{2}\pi i t}}\,dt =$

$= 2\pi i - \frac{1}{2}\pi i \underbrace{\int_0^1 1\,dt}_{=1}$

$= 2\pi i - \frac{1}{2}\pi i = \frac{3}{2} \cdot \pi i \Rightarrow$ $f\left(e^{\frac{3}{2}\pi i}\right) = \frac{3}{2}\pi i$

Braun SoSe 2020 Funktionentheorie Klausur

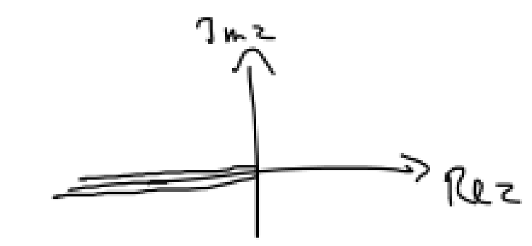
1) Es sei log der Hauptzweig des Logarithmus

(a) Bestimmen Sie $\log \frac{1}{e^2}$ und $\log e^{\frac{3}{2}\pi i}$.

(b) Es sei $G = \mathbb{C} \setminus i[0,\infty[$ und es sei $f: G \to \mathbb{C}$ der durch $f(1) = 2\pi i$ gegebene Zweig des Logarithmus.

Bestimmen Sie $f(\frac{1}{e^2})$ und $f(e^{\frac{3}{2}\pi i})$.

(c) Bestimmen Sie alle $z \in G \setminus]-\infty,0]$, in denen log und f übereinstimmen.

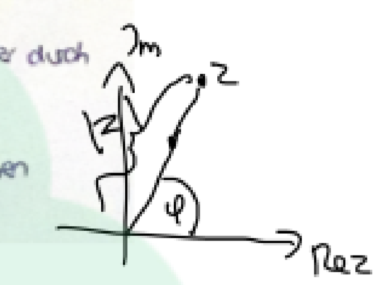


$\log: \mathbb{C} \setminus]-\infty,0] \to \mathbb{C}$

$\underbrace{z}_{|z| \cdot e^{i\varphi}} \mapsto \log(|z| \cdot e^{i\varphi}) = \ln(|z|) + \log(e^{i\varphi})$, $\varphi \in]0, 2\pi[$

$= \ln(|z|) + i\varphi$

$\ln(a \cdot b) = \ln(a) + \ln(b)$

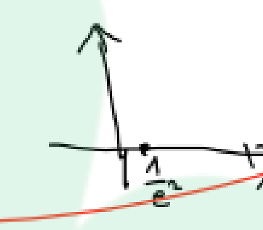


a) $\log \frac{1}{e^2} =$

$= \log\left(\overset{|z|}{\frac{1}{e^2}} \cdot e^{i \cdot \overset{\varphi}{0}}\right) =$

$= \ln\left(\frac{1}{e^2}\right) + i \cdot 0$

$= \ln\left(\frac{1}{e^2}\right) = \ln\left(e^{-2}\right) = -2$



$-1 = 1 \cdot e^{i\pi}$

$5 = 5 \cdot e^{i0}$

Hauptzweig komplexer Logarithmus

$\log z = \log|z| + i\varphi$

reeller Logarithmus

$\log\left(e^{\frac{3}{2}\pi i}\right) = \log\left(\overset{|z|}{1} \cdot e^{\overset{\varphi}{\frac{3}{2}\pi} i}\right)$

$= \ln(1) + i \cdot \frac{3}{2}\pi = 0 + \frac{3}{2}\pi i = \frac{3}{2}\pi i$

$\ln(1) = \ln(e^0) = 0$