

\hat{A} operator
 $\forall \psi, \phi :$
 $\langle \hat{A}^\dagger \psi, \phi \rangle = \langle \psi, \hat{A} \phi \rangle$

Problem 6.1: Orthogonality of bound state solutions
 Two states $\varphi_1(x)$ and $\varphi_2(x)$ are bound state solutions (real functions) of the Schrödinger equation, with energy eigenvalues E_1 and E_2 respectively, where $E_1 \neq E_2$. Show that $\varphi_1(x)$ and $\varphi_2(x)$ are orthogonal. (4 points)

A operator δ eigenvalue of A \Leftrightarrow
 $\exists \psi \neq 0$ such that
 $A \cdot \psi = \delta \cdot \psi$
 eigenwert

Jeder hermitesche Operator \hat{A} hat nur reelle Eigenwerte.

$\langle \cdot, \cdot \rangle$ ist positiv definit
 $\langle \psi, \psi \rangle = 0 \Leftrightarrow \psi = 0$

H ist hermitesch,
 d.h. $H = H^\dagger$, d.h.

$H \varphi_1 = E_1 \cdot \varphi_1$
 $H \varphi_2 = E_2 \cdot \varphi_2$

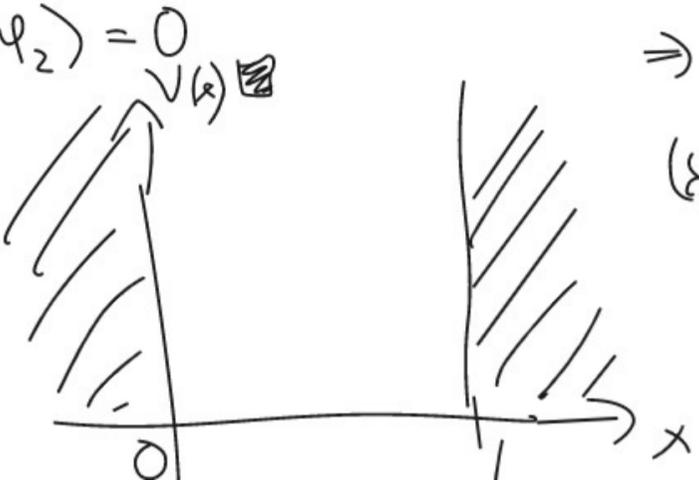
Zu zeigen: $\langle \varphi_1, \varphi_2 \rangle = 0$

H hermitesch

$E_1 \langle \varphi_1, \varphi_2 \rangle = E_1^* \langle \varphi_1, \varphi_2 \rangle = \langle E_1 \varphi_1, \varphi_2 \rangle = \langle H \varphi_1, \varphi_2 \rangle \stackrel{H \text{ hermitesch}}{=} \langle \varphi_1, H \varphi_2 \rangle = \langle \varphi_1, E_2 \varphi_2 \rangle = E_2 \langle \varphi_1, \varphi_2 \rangle$

$\Rightarrow E_1 \langle \varphi_1, \varphi_2 \rangle = E_2 \langle \varphi_1, \varphi_2 \rangle \quad | - E_2 \langle \varphi_1, \varphi_2 \rangle$

$(E_1 - E_2) \cdot \langle \varphi_1, \varphi_2 \rangle = 0$
 $\neq 0 \Rightarrow \langle \varphi_1, \varphi_2 \rangle = 0$



Problem 6.2: Particle in an asymmetric infinite potential well

Consider a particle of mass m in a potential field given by

$$V(x) = \begin{cases} 0, & 0 < x < L, \\ \infty, & x < 0 \vee x > L. \end{cases}$$

At the initial time $t = 0$ the wave function of the particle is given by

Beweis: Sei $\delta \in \mathbb{C}$ ein Eigenwert von \hat{A} , d.h. wir finden ein $\psi \neq 0$ sodass $\hat{A} \psi = \delta \cdot \psi$. \hat{A} hermitesch

$\delta^* \langle \psi, \psi \rangle = \langle \delta \psi, \psi \rangle = \langle \hat{A} \psi, \psi \rangle \stackrel{\hat{A} \text{ hermitesch}}{=} \langle \psi, \hat{A} \psi \rangle = \langle \psi, \delta \psi \rangle = \delta \langle \psi, \psi \rangle$

$\Rightarrow \delta^* \langle \psi, \psi \rangle = \delta \cdot \langle \psi, \psi \rangle \quad | - \delta \cdot \langle \psi, \psi \rangle$

$(\delta^* - \delta) \cdot \underbrace{\langle \psi, \psi \rangle}_{\neq 0, \text{ weil } \psi \neq 0} = 0$

$\delta^* - \delta = 0$
 $\Rightarrow \delta^* = \delta \Rightarrow \delta \in \mathbb{R}$

$\langle \varphi_1, \varphi_2 \rangle = \int_{-\infty}^{\infty} \varphi_1^* \varphi_2 dx = \int_{-\infty}^{\infty} \alpha^* \varphi_1^* \varphi_2 = \alpha^* \langle \varphi_1, \varphi_2 \rangle$

$\forall a \in \mathbb{R}: a^* = a$

$\alpha \in \mathbb{C}$
 $\langle \alpha \varphi_1, \varphi_2 \rangle = \alpha^* \langle \varphi_1, \varphi_2 \rangle$
 $\langle \varphi_1, \alpha \varphi_2 \rangle = \alpha \langle \varphi_1, \varphi_2 \rangle$